Physics 716 Problem set 3 Spring 2018

Due March 1, 2018

Reading

Peskin & Schroeder Sections 9.2-9.6

Suggested further reading: Weinberg Volume 1, Sections 9.4-9.5

Problems

1. *Gauge slicing with a toy model.* Consider the integral,

$$I(a) = \int d^2x \, e^{-f(r)} \,, \tag{1}$$

where $f(r) = ar^2/2$. Here rotational symmetry is an analog of gauge symmetry. Recall that we inserted the identity,

$$1 = \int d\phi \, \delta\big[g(\boldsymbol{r}_\phi)\big] \left| \frac{\partial g(\boldsymbol{r}_\phi)}{\partial \phi} \right| \,, \tag{2}$$

with $g(\boldsymbol{r}) = \theta - \theta_0$, to isolate one of the equivalent "gauge slices" (at $\theta = \theta_0$). Perform the analysis using instead

$$g(\boldsymbol{r}) = \theta - \frac{\pi}{2} e^{-r^2} \,. \tag{3}$$

2. *Grassmann variables.* Consider the integral, for a Grassman algebra generated by independent θ_i and $\bar\theta_i$,

$$I(a) = \int d\theta_1 d\bar\theta_1 \ldots d\theta_n d\bar\theta_n \exp\left[\sum_{i,j=1}^{N} \bar\theta_i a_{ij} \theta_j \right] . \tag{4}$$

Compute $I(a)$ in two ways, first by explicitly expanding the exponential, and second by considering the variable change $\theta'_i = \sum_j a_{ij}\theta_j$.

2. *Gratuitous ghosts.* Consider a free scalar lagrangian,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 \,. \tag{5}$$

a) Argue that $\int [d\phi] \phi(x) e^{iS(\phi)} = 0$.

b) Now consider the change of variables,

$$\phi = B + gB^2 \,, \tag{6}$$

where g is a small coupling. Find the lagrangian in terms of B to first order in g. Find the Jacobian for the change of variables,

$$[d\phi] = [dB] \det \frac{\partial \phi}{\partial B} \,, \tag{7}$$

and write the determinant as a path integral over ghosts.

c) Find the diagrams that contribute to the quantity

$$\int [dB][d\bar\eta][d\eta](B + gB^2) \exp[iS(B, \bar\eta, \eta)] \,, \tag{8}$$

at first order in g. Compute the diagrams and show that the answer from part (a) is recovered.

3. *Gauge fixing.* Use the Fadeev Popov ansatz to find the Feynman rules for QED using gauge-fixing function $f(A) = n \cdot A$, where n^μ is a light-like vector, $n^2 = 0$. Is the resulting theory Lorentz invariant?