Physics 716 Problem set 2 Spring 2018

Due March 8

Reading

Peskin & Schroeder Sections 9.1

Suggested further reading: Weinberg Volume 1, Sections 9.1-9.3

Problems

1. Consider a non-relativistic particle of mass m, moving along the x axis in a potential $V(x) = m\omega^2 x^2/2$. Use path integral methods to find the probability to find the particle between x_1 and $x_1 + dx_1$ at time t_1 if the particle is at x_0 at time t_0. Does your answer make sense in limits such as $\omega \to 0$, $t_1 - t_0 \to 0$, $x_1 - x_0 \to 0$?

2. Recall that for a particle of unit mass in a simple harmonic oscillator potential (cf. problem 1), the ground state satisifes

$$a|\text{g.s.}\rangle = 0\,, \tag{1}$$

where the Heisenberg picture operators are given by

$$x(t) = \frac{1}{\sqrt{2\omega}}\left(a + a^\dagger\right)\,, \quad p(t) = -i\sqrt{\frac{\omega}{2}}\left(a - a^\dagger\right)\,. \tag{2}$$

a) Solve for a as a function of x and p. Multiply (1) by $\langle x|$ and find the resulting equation for the position space wavefunction, $\langle x|\text{g.s.}\rangle$. Solve this equation to obtain the usual solution to the harmonic oscillator ground state. Find the first excited state by similar methods, applying $a^\dagger$ expressed in terms of x and d/dx to the ground state.

b) In class we solved for the vacuum wavefunction for a scalar field of mass m (in the space of coordinates $\phi(\mathbf{x})$, $\mathbf{x}$ running over space), $\langle\phi(\mathbf{x}); \pm\infty|\text{g.s.}; \pm\infty\rangle$. Extending (a) to the field theory case, find the wavefuntion corresponding to single particle states.