Physics 716 Problem set 1 Spring 2018

Due Jan. 25

Reading

Peskin & Schroeder Section 3.1 and Problem 3.1(a); Sections 4.1-4.4

Problems

1. **Fermion lagrangians.** Consider two such fermions, ψ_L and ψ'_L, and the lagrangian

$$\mathcal{L} = \psi_L^\dagger i\bar{\sigma}\cdot\partial\psi_L + \psi_L'^\dagger i\bar{\sigma}\cdot\partial\psi'_L - m\left(\psi_L^\dagger i\sigma^2\psi_L'^* - \psi_L'^T i\sigma^2\psi_L\right), \tag{1}$$

where $\sigma^\mu = (1, \boldsymbol{\sigma})$ and $\bar{\sigma}^\mu = (1, -\boldsymbol{\sigma})$.

a) *Lorentz symmetry.* Recall the transformation law for the elementary (left-handed) fermion,

$$\psi_L(x) \to \Lambda_L\psi_L(\Lambda^{-1}x), \tag{2}$$

where $\Lambda_L = e^{i\left(\boldsymbol{\theta}\cdot\boldsymbol{J}+\boldsymbol{\eta}\cdot\boldsymbol{K}\right)}$, with $\boldsymbol{\theta}$ and $\boldsymbol{\eta}$ the rotation and boost parameters of a given Lorentz transformation, and $J = \boldsymbol{\sigma}/2$, $K = i\boldsymbol{\sigma}/2$ the rotation and boost generators. Show that $\mathcal{L}$ is invariant when ψ_L and ψ'_L transform according to (2).

b) *Electric charge symmetry.* Show that $\mathcal{L}$ is invariant when $\psi_L \to e^{i\epsilon}\psi_L$ and $\psi'_L \to e^{-i\epsilon}\psi'_L$, for arbitrary real ϵ.

c) *Charge conjugation symmetry.* Show that $\mathcal{L}$ is invariant when $\psi_L \leftrightarrow e^{i\phi_1}\psi'_L$. What are the constraints on ϕ_1?

d) *Parity symmetry.* Show that $\mathcal{L}$ is invariant when $\boldsymbol{x} \to -\boldsymbol{x}$ and $\psi_L \leftrightarrow e^{i\phi_2}\big[-i\sigma^2\psi_L'^*\big]$. What are the constraints on ϕ_2?

e) *Time reversal symmetry.* Show that $\mathcal{L}$ is invariant when $t \to -t$, $\psi_L \to e^{i\phi_3}\big[-i\sigma^2\psi_L\big]$ and $\psi_L \to e^{i\phi_4}\big[-i\sigma^2\psi'_L\big]$. What are the constraints on ϕ_3, ϕ_4?

e) *Dirac notation.* Consider the identifications

$$\psi_R = i\sigma^2\psi_L'^*, \quad \Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}, \quad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}. \tag{3}$$

Express the lagrangian (1) in terms of Ψ and γ^μ. Express the transformation laws in (a)-(d) in terms of Ψ and γ^μ.

2. **Wick's theorem and Feynman rules.** An essential result for perturbative quantum field theory is the relation

$$\langle\text{vac}|T\{\phi(x_1)\phi(x_2)\dots\}|\text{vac}\rangle = \frac{\langle 0|T\{\phi_I(x_1)\phi_I(x_2)\dots\exp\left[i\int d^dx\mathcal{L}_I(x)\right]\}|0\rangle}{\langle 0|T\{\exp\left[i\int d^dx\mathcal{L}_I(x)\right]\}|0\rangle}, \tag{4}$$

where $|0\rangle$ is the free vacuum, $\phi_I(x)$ is the interaction picture field expressed in terms of creation and annihilation operators,

$$\phi_I(x) = \int\frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2p^0}}\left(e^{-ip\cdot x}a_{\boldsymbol{p}} + e^{ip\cdot x}a_{\boldsymbol{p}}^\dagger\right)\Big|_{p^0=E_{\boldsymbol{p}}}, \tag{5}$$

and $\mathcal{L}_I(x)$ is the interaction lagrangian. (Scattering amplitudes are obtained from this formula using LSZ reduction.)

a) *Wick's theorem.* Consider the Lagrangian,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - g\phi^3 \,. \tag{6}$$

Use Wick's theorem to compute the RHS (numerator and denominator) of (4) to second order in g. You may leave momentum space integrals over d^4p unevaluated.

b) *Feynman rules.* Set $x_1 = x$ and $x_2 = 0$, and consider the Fourier transform of the result in (a) with respect to x. Interpret the result in terms of Feynman diagrams. For the connected diagrams, identify any infrared or ultraviolet divergences in the momentum space integrals.